

Benefits of Sigma-Delta ADCs

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Due to the complexity of designing baseband antialiasing filters with sharp transitions, it is beneficial to use sigma-delta ($\Sigma\Delta$) ADCs for data conversions in communication (voiceband and audio) applications. In order to appreciate this benefit, consider the following single tone signal using a standard ADC, sampled in the time domain.

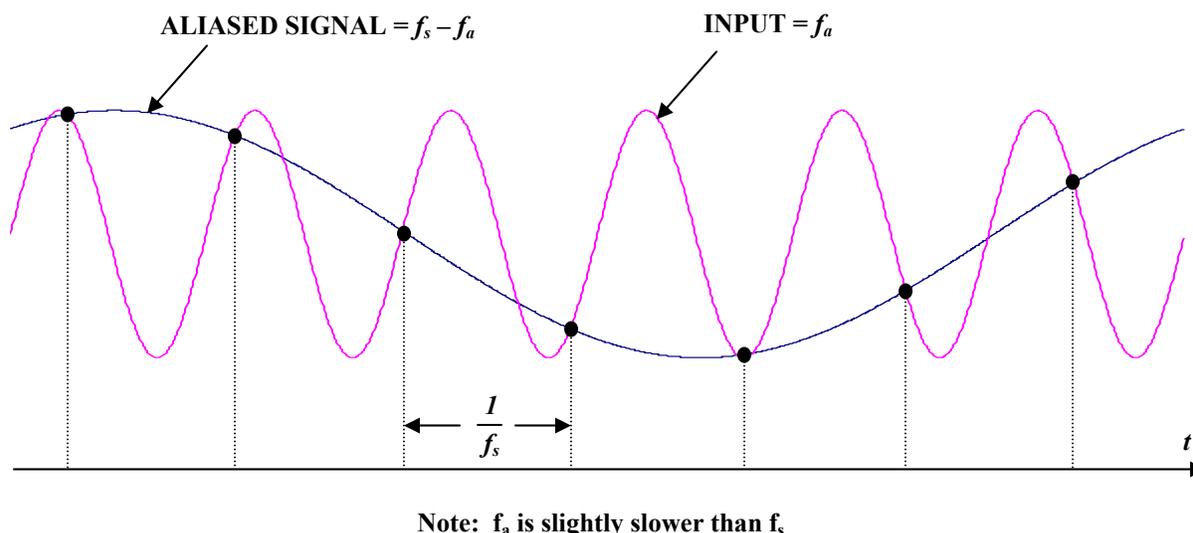


Figure 1

In order to maintain the concepts of *discrete time sampling* and *finite amplitude resolution due to quantization*, continuous analog data must be sampled at discrete intervals of $t_s = 1/f_s$. This interval must be carefully chosen to insure accurate representation of the original signal. The more samples taken for a particular signal (using faster sampling rates), the more accurate the digital representation will be. Additionally, if fewer samples are taken (using lower sampling rates), critical information about the signal may be lost. Therefore, it is important to abide to Nyquist's criteria which states that

- A signal with a *bandwidth* f_a must be sampled at a rate of $f_s > 2f_a$ or information about the signal will be lost.

The concept of *aliasing* occurs whenever this criteria is broken where $f_s < 2f_a$. In Figure 1 above, f_s is only slightly faster than the analog input frequency f_a , which violates Nyquist's criteria. An *aliased* sine wave at a lower frequency equal to $f_s - f_a$ is produced. This adds non-harmonically related frequency components to the sampled signal which cannot be distinguished or removed from the signal of interest.

The corresponding frequency domain representation for Figure 1 is shown in Figure 2. Notice the *aliases* that show up, especially $f_s - f_a$ within the bandwidth of f_a which cannot be filtered or removed.

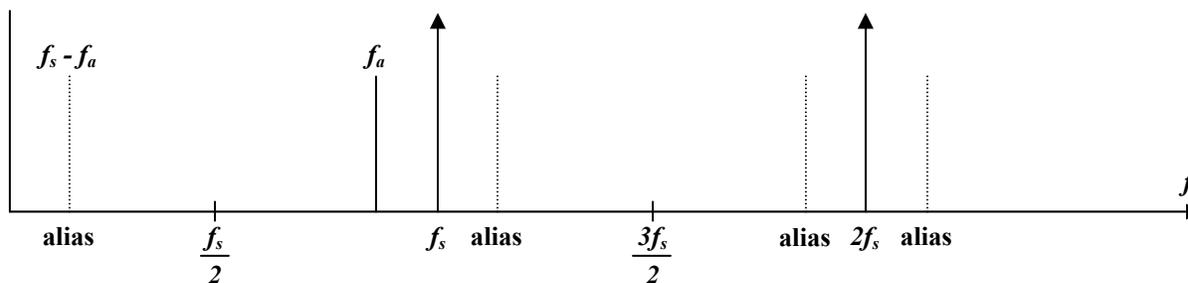


Figure 2

Now consider the case of a single tone signal of frequency f_a , sampled at frequency f_s , using an ideal impulse sampler. Assume that $f_s > f_a$ as shown in Figure 3. The output of this signal in the frequency domain shows that there are *aliases* of the original signal around every K multiple of f_s . That is, at frequencies equal to $|\pm Kf_s \pm f_a|$, where $K=1,2,3,4,\dots$

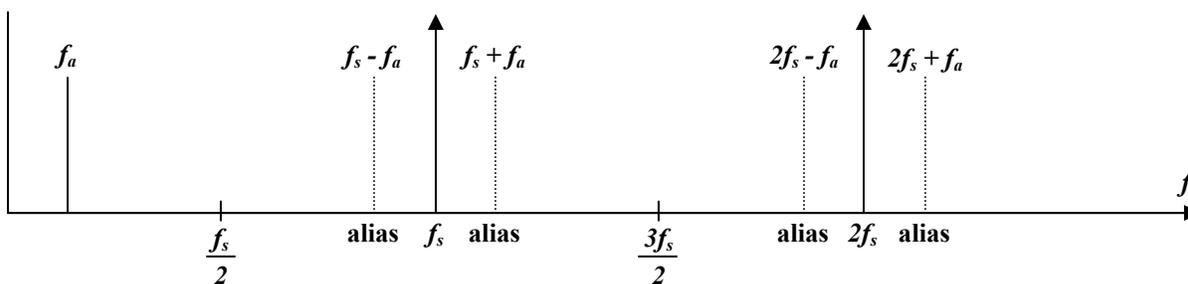


Figure 3

The Nyquist bandwidth is defined as the frequency spectrum from DC to $f_s/2$. Although the signal in Figure 2 is outside the Nyquist bandwidth or the first Nyquist zone, its alias $f_s - f_a$, falls inside. As mentioned before, this is not filterable. However, in the case where $f_s > f_a$ in Figure 3, it is quite clear that an unwanted signal appears at any of the image frequencies of f_a , it will also occur at f_a , producing a spurious frequency component in that first Nyquist zone. This is very similar to analog mixing components in communication applications, implying that some filtering prior to the sampler or ADC is required to remove frequency components which are outside the Nyquist bandwidth, but the aliased components lie within it. The filter performance and complexity will depend on how close the out-of-band signal is to $f_s/2$ and the amount of attenuation required.

The use of baseband antialiasing filters provides the proper filtering required to block unwanted frequency components (either signal or noise) outside the Nyquist bandwidth that alias back into the first Nyquist zone. Therefore, properly specifying the baseband antialiasing filter is very important. The initial step to doing so is to know the characteristics of the signal being sampled by assuming that the highest frequency of interest is f_a . The baseband antialiasing filter will allow passage of signals from DC to f_a while attenuating signals above f_a .

Assume that the corner frequency of the filter is chosen to be equal to f_a . Keep in mind that there is a finite transition (minimum to maximum attenuation) for this filter from f_a to $f_s - f_a$. This limits the dynamic range of the system. Ideally a brick wall low-pass filter (at the corner frequency of f_a) would be desired; however, this filter would be very difficult to design and is unrealizable within the constraints of the system cost and performance. Therefore, the antialiasing filter transition is determined by three attributes: (1) the corner frequency f_a , (2) the stopband frequency $f_s - f_a$, (3) and the dynamic range (DR), given by the desired stopband attenuation as shown in Figure 4a below.

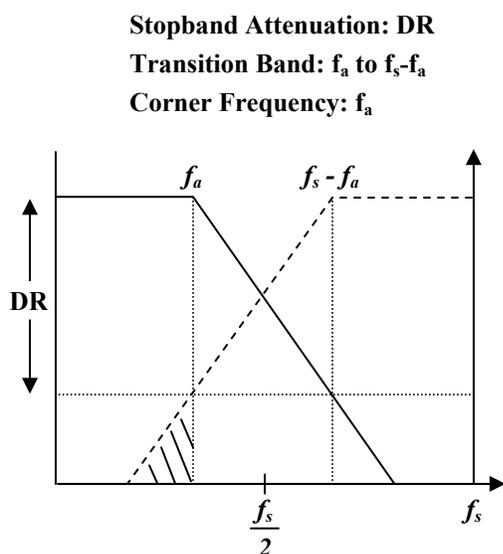


Figure 4a

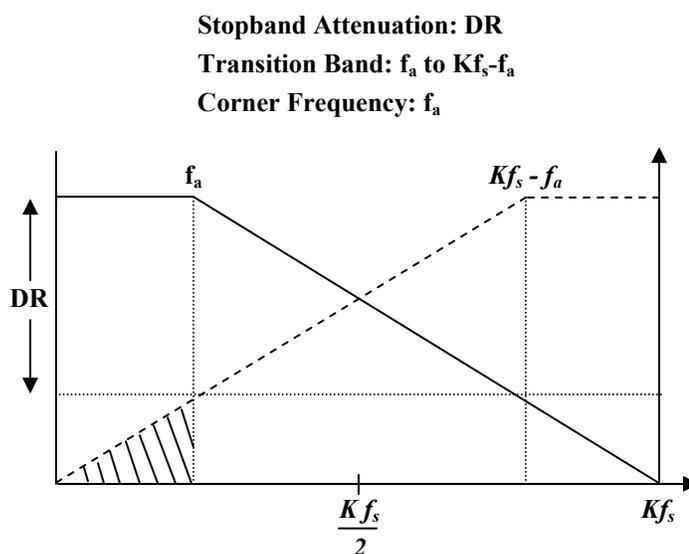


Figure 4b

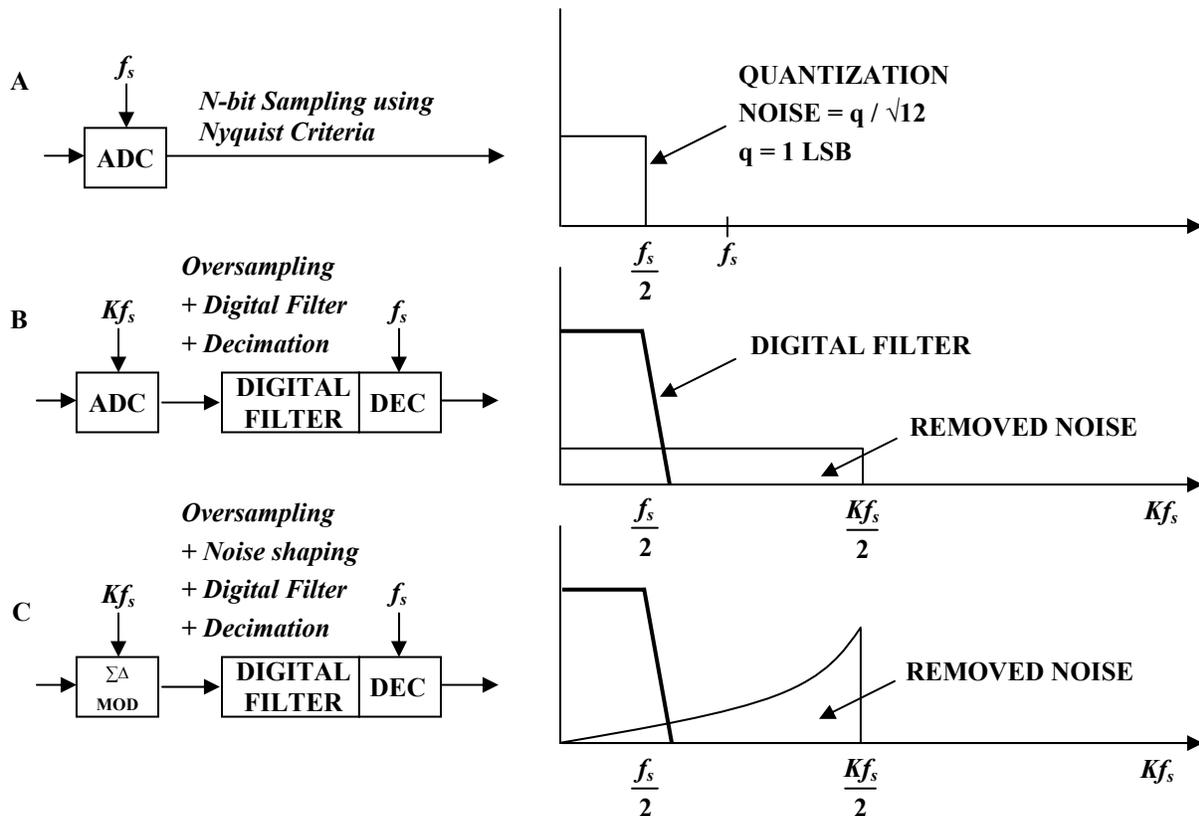
Filters become more complex and difficult to design as the transition band becomes sharper and sharper, while the corner frequency and the dynamic range remain the same. As we can see in Figure 4b above, the sharpness of the antialiasing transition band can be traded off against the ADC sampling frequency. Choosing a higher sampling rate, or *oversampling*, relaxes the requirement on the sharpness of the transition band (hence, the complexity of it) at the expense of using a faster ADC. Additionally the data would be processed at a faster rate. The wider transition band (f_a to $Kf_s - f_a$) in Figure 4b makes the filter design easier than in the case of Figure 4a.

Sigma-delta ($\Sigma\Delta$) ADCs are inherently oversampling converters, and the resulting relaxation in the baseband antialiasing filter requirements is an added benefit for this type of architecture. They are now used in many applications where a low-cost, low-bandwidth (voiceband and audio), low-power, and high resolution ADC is required. The basic components of a $\Sigma\Delta$ ADC contain simple analog electronics (a comparator, voltage reference, a switch, one or more integrators, analog summing circuits, and a DSP). The DSP acts as a mathematical filter, generally a low pass filter. To understand how a $\Sigma\Delta$ ADC works, it is advised to be familiar with concepts of *oversampling*, *quantization noise shaping*, *digital filtering*, and *decimation*.

To touch on these basic concepts, consider the technique of *oversampling* using frequency domain analysis. A DC conversion with a quantization error of up to $\frac{1}{2}$ LSB has *quantization noise*. Figure 5a shows a “perfect” classical N-bit sampling ADC has an RMS quantization noise of $q/\sqrt{12}$ uniformly distributed within the Nyquist band of DC to $f_s/2$ (where q is the value of an LSB and f_s is the sampling rate). The SNR with a full-scale sine wave input will be $(6.02N + 1.76)$ dB. If the sampling ADC is less than perfect, and its noise is greater than its theoretical minimum quantization noise, then its *effective* resolution will be less than N-bits. The error in the quantization noise is random and uncorrelated with the perfect representation signal. The actual representation is given by the equation (often referred to as Effective Number of Bits or ENOB).

$$ENOB = \frac{SNR - 1.76dB}{6.02dB} \tag{1}$$

Choosing a higher sampling rate of Kf_s , the RMS quantization noise remains at $q/\sqrt{12}$, however the noise is now distributed over a wider transition band of DC to $Kf_s/2$ as in figure 5b. Applying a digital low pass filter (LPF) to the output will remove the quantization noise without affecting the desired signal, therefore ENOB is improved. We have accomplished a high resolution A/D conversion with a low resolution ADC, while relaxing the requirements for the input analog antialiasing filter.



Figures 5a-c

The output data rate can now be reduced lower than the original sampling rate of Kf_s because the bandwidth of the digital LPF on the output. The Nyquist criterion is still satisfied. The slower data rate is achieved by a process called *decimation* by a factor of M . This process passes every M th bit to the output, while discarding the remainder. The factor M can be any integer value, provided that the output data rate is greater than twice the signal bandwidth. No loss of information will result from the decimation process.

Resolution can be improved with oversampling, however, we must over-sample by a factor of 2^{2N} in order to obtain an increase in resolution of N -bits. From figure 5c, we can see that the $\Sigma\Delta$ converter not only limits the signal passband, but also shapes the quantization noise so that most of it falls outside of the passband. Therefore, the $\Sigma\Delta$ converter does not require such a high over-sampling ratio.

The first-order $\Sigma\Delta$ modulator shown in Figure 6, is comprised of a 1-bit ADC (typically known as a comparator) driven by the output of an integrator that is fed with an input signal summed with the output of a 1-bit DAC fed from the ADC output. Adding a digital low pass filter (LPF) and a decimator to the digital output will create a $\Sigma\Delta$ ADC. The $\Sigma\Delta$ modulator shapes the quantization noise so that it lies above the passband of the digital output filter. Therefore, the ENOB is much larger than just using oversampling.

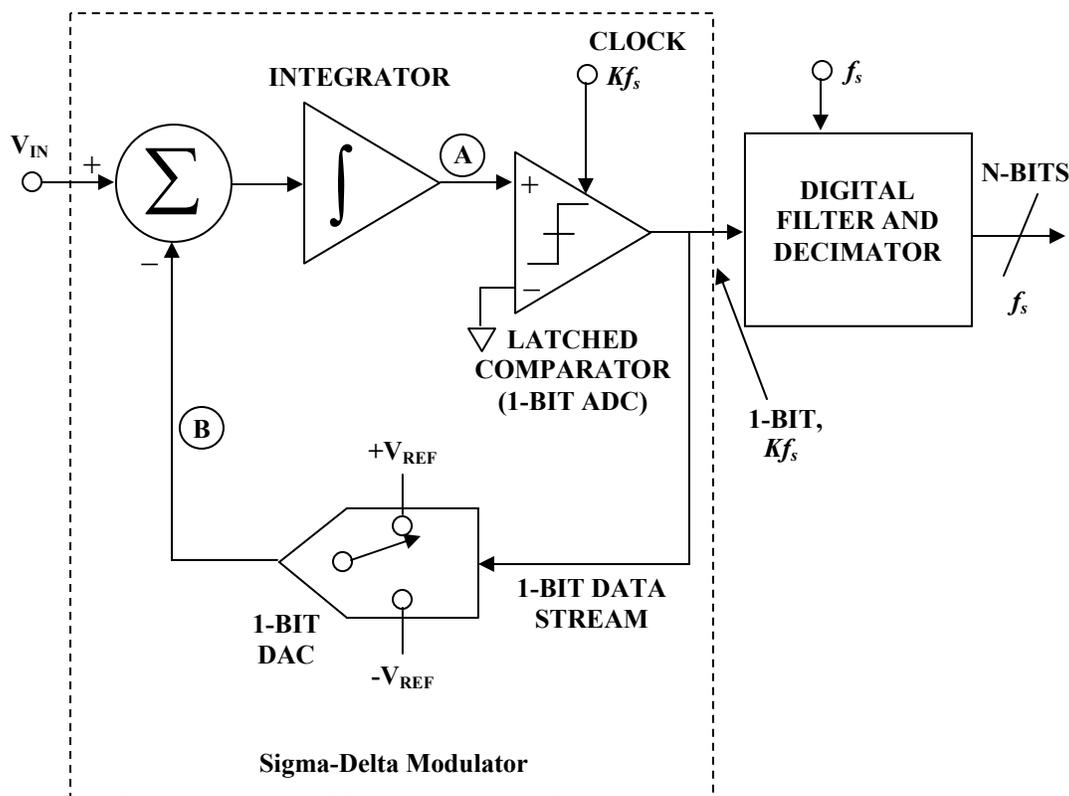


Figure 6, First Order Sigma-Delta ($\Sigma\Delta$) ADC

From Figure 6 above, assume a DC input at V_{IN} . The integrator constantly ramps up or down at node A, which feeds the comparator or 1-bit ADC. The output of the 1-bit ADC controls the *ones-density* that is fed back through a 1-bit DAC which creates an average output voltage at node B at the rate of Kf_s . This average voltage is then forced to be equal to V_{IN} by the summing point. As the input signal increases towards $+V_{REF}$, the number of “ones” in the serial bit stream increases, and the number of “zeros” in the serial bit stream decreases. Conversely, as the input signal goes negative towards $-V_{REF}$, the number of “zeros” in the serial bit stream increases, and the number of “ones” decreases. Simply, the average value of the input voltage is contained in the serial bit stream of the comparator running at Kf_s . The digital filter and decimator, running at a rate of f_s , processes the serial bit stream and produces the final output data.

REFERENCES

1. Walt Kestler and James M. Bryant, *Mixed Signal and DSP Design Techniques*, Analog Devices, Inc., 2003.
2. Brian A. Lowe, Jr., *The Fundamentals of Mixed Signal Testing*, Soft Test, Inc., 1999.

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